CHAPTER 12: SUPPLEMENT 5

Methods Of Calculating Equity Yield

C12.S5.1 CALCULATING THE YIELD GREEKS

C12.S5.1.1 Interest Rate Sensitivity

Duration and Rho (ρ)—There are multiple ways to estimate the interest rate sensitivity of bonds. Since straight bonds are essentially a collection of fixed future cash flows, the simplest method, which is called Macaulay Duration, is to calculate the weighted average time to each cash flow, divided by the current price (which represents what would be a negative cash flow if one were to purchase the bond today).

$$Macaulay\ Duration=\frac{\int_{t=1}^{n}\frac{t\times C}{(1+y)^{t}}+\frac{n\times M}{(1+y)^{n}}}{\text{Current Bond Price}} \tag{1}$$

where: C = periodic coupon payment, y = periodic yield, M = the bond's maturity value, n = duration of bond in periods of time.

However, this does not give the precise interest rate sensitivity for straight bonds. In order to obtain rate sensitivity, one must modify the formula to account for the yield to maturity, hence the second method is called modified duration, and is calculated as follows:

$$Modified\ Duration = \frac{Macauley\ Duration}{(1+\text{YTM}/\text{k})} \tag{2}$$

Where: YTM = yield to maturity, k = number of coupon periods per year

The key observation for our purposes is that higher rates lead to losses on a straight bond.

While modified duration works well for calculating the interest-rate sensitivity of straight bonds, modified duration ignores the interest rate sensitivity that convertibles derive from the embedded option to convert into common shares. To calculate convertible duration, or rho, one must incorporate the rate sensitivity of the embedded call option.

Recall from Chapter 8 that an option's value has two basic parts, the expected intrinsic value at maturity (which is essentially the delta), and the present value of the "loan" needed to obtain the delta. Rho is focused on the latter, and is calculated as follows:

$$\text{Rho}(\rho) = \mathrm{Kte}^{-\mathrm{rt}}\mathrm{N}(\mathrm{d}_2) \quad (3)$$

Where

$$\mathrm{d}_1 = \frac{\ln(\mathrm{S}/\mathrm{K}) + \mathrm{t}\times(\mathrm{r}-\mathrm{q}+\frac{\sigma^2}{2})}{\sigma\times\sqrt{\mathrm{t}}}$$

$$\mathrm{d}_2 = \mathrm{d}_1 - \sigma\times\sqrt{\mathrm{t}}$$

$$\mathrm{N}(\mathrm{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\mathrm{x}} \mathrm{e}^{\frac{-\mathrm{d}_1{}^2}{2}}$$

The key observation is that higher rates *increase* the value of rho for an option, which is the opposite effect of higher rates on a straight bond. This is because the above formula assumes that the underlying stock price drifts higher at the risk free rate minus the dividend yield plus one half of the variance $(\mathrm{r}-\mathrm{q}+\frac{\sigma^2}{2})$. A higher figure for the risk- free rate increases the likelihood of conversion. Note that this is distinct from an increase in credit spreads, which does not affect option value.

So let's consider the effect of convertibility on the interest rate sensitivity of a bond by comparing a straight (nonconvertible) bond with a convertible bond that has the exact same maturity and coupon payment dates. Say we have a straight bond that matures in 10 years and pays 5% coupons annually. We also have a convertible bond that matures in 10 years and pays 3% coupons annually but is also convertible and has a 40% conversion premium at issuance. Let's assume the stock has a volatility of 35%, risk free rates are 2%, and both bonds are priced at par.

Using the formula for modified duration, the interest rate sensitivity of both the straight bond and the bond portion of the convertible would be

$$\frac{\int_{\mathrm{t}=1}^{10}\frac{\mathrm{t}\times 50}{(1+.05)^{\mathrm{t}}}+\frac{10\times 1000}{(1+.05)^{10}}}{1000} = 8.108$$

$$\text{Modified Duration} = \frac{8.108}{(1+0.05)} = 7.722$$

Using the formula for rho, the interest rate sensitivity of the call option would be

$$\rho_{\text{call}} = \mathrm{K}\times\mathrm{t}\times\mathrm{e}^{-\mathrm{rt}}\times\mathrm{N}(\mathrm{d}2)$$

$$\mathrm{d}_2 = \frac{\boldsymbol{LN}\left(\frac{\boldsymbol{S}}{\boldsymbol{K}}\right)+\boldsymbol{t}\times(\boldsymbol{r}-\boldsymbol{q}+\frac{\boldsymbol{\sigma}^2}{2})}{\boldsymbol{\sigma}\sqrt{\boldsymbol{t}}} - \boldsymbol{\sigma}\sqrt{\boldsymbol{t}}$$

$$\mathrm{d}_2 = \frac{\boldsymbol{LN}\left(\frac{\mathbf{100}}{\mathbf{140}}\right)+\mathbf{10}\times(0.\mathbf{02}-\mathbf{0}+\frac{0.35^{\mathbf{2}}}{2})}{0.\mathbf{35}\times\sqrt{\mathbf{10}}} - 0.35\times\sqrt{10} = -0.6767$$

$$\mathrm{N}(\mathrm{d}_2) = \mathrm{N}(-0.6767) = 0.2493$$

$$\rho_{\text{call}} = \frac{\mathbf{1}}{\mathbf{100}}\times 140\times 10\times\mathrm{e}^{-0.02\times 10}\times 0.2493$$

$$\rho_{\text{call}} = 2.858$$

This means the value of the underlying option increases by \$2.858 per 1 percentage point increase in rates. This is different from duration, which enables calculation of the percent increase/decrease in bond value given a percent change in rates. In order to determine convertible interest rate sensitivity, we need to make sure our units of measurement are identical before calculating a weighted average.

Using the Black Scholes formula the “per contract” (i.e., an option for 100 shares) of the call option is \$38.07, so the percent increase in the call option is $\frac{\$2.858}{\$38.07} = 0.07513$ or 7.513%

Therefore the interest-rate sensitivity of the convertible would be…

$$\rho_{\text{convertible}} = \left(\frac{845.57}{1000} \times 8.245\right) + \left(\frac{154.43}{1000} \times -7.513\right) = 5.811 \text{ or } 5.811\%$$

We can then see that, despite the similarity of term and coupon, the rate sensitivity of the straight bond is 32.89% higher than that of the convertible.

$$\rho_{\text{bond}} / \rho_{\text{convertible}} = \frac{7.722}{5.811} = 1.329$$

If the stock price were to advance by 40% in the first two years, the conversion premium would be reduced to 0, and the convertible rho would drop significantly.

Rate sensitivity in the straight bond would be reduced due to the passage of two years to maturity, to 6.46. Rate sensitivity in the convertible would be reduced due to both the passage of time, and a significant increase in the value of the option, to 4.31. Therefore the relative rate sensitivity would increase from 1.329 to 1.499.

C12.S5.1.2 Interest Rate Convexity (q)

The rate sensitivity of bonds is not static; it changes as rates fluctuate. To illustrate this rate sensitivity, let’s consider the straight 10-year bond with a 5% coupon as well as a bond with a 4% coupon and one with a 6% coupon.

We can immediately see that Macaulay duration will be higher for the bond with a 4% coupon since less of the total cash flow of the bond occurs prior to maturity (26% instead of 30%) resulting in Macaulay duration of 8.628 instead of 8.416. We can also see that the duration should be smaller for the bond with the 6% coupon since a greater portion of the total cash flow of the bond occurs prior to maturity (34% instead of 30%) resulting in Macaulay duration of 8.230 instead of 8.416. Similarly, modified duration is higher in the 4% bond (8.306 instead of 8.025) and lower in the 6% bond (7.774 instead of 8.025).

C12.C5.T5.01 Duration and Convexity

	Cash Flows			Relative Wt. of Cash Flows			Wt. * Years		
	Bond A	Bond B	Bond C	Bond A	Bond B	Bond C	Bond A	Bond B	Bond C
Year	5% Coupon	4% Coupon	6% Coupon	5% Coupon	4% Coupon	6% Coupon	5% Coupon	4% Coupon	6% Coupon
Price	101	101	101						
1	5	4	6	3.33%	2.86%	3.75%	0.033	0.029	0.038
2	5	4	6	3.33%	2.86%	3.75%	0.067	0.057	0.075
3	5	4	6	3.33%	2.86%	3.75%	0.100	0.086	0.113
4	5	4	6	3.33%	2.86%	3.75%	0.133	0.114	0.150
5	5	4	6	3.33%	2.86%	3.75%	0.167	0.143	0.188
6	5	4	6	3.33%	2.86%	3.75%	0.200	0.171	0.225
7	5	4	6	3.33%	2.86%	3.75%	0.233	0.200	0.263
8	5	4	6	3.33%	2.86%	3.75%	0.267	0.229	0.300
9	5	4	6	3.33%	2.86%	3.75%	0.300	0.257	0.338
10	105	104	106	70.00%	74.29%	66.25%	7.000	7.429	6.625
Total Cashflow	150	140	160		Maculay duration		8.416	8.628	8.230
Prior to Maturity	45	36	54		Yield to Maturity		4.876	3.881	5.871
% of Total Prior	30%	26%	34%		Modified duration		8.025	8.306	7.774
					Difference from A			0.281	(0.251)
					Convexity Approximation		0.266		

Source: Advent Capital Management, LLC

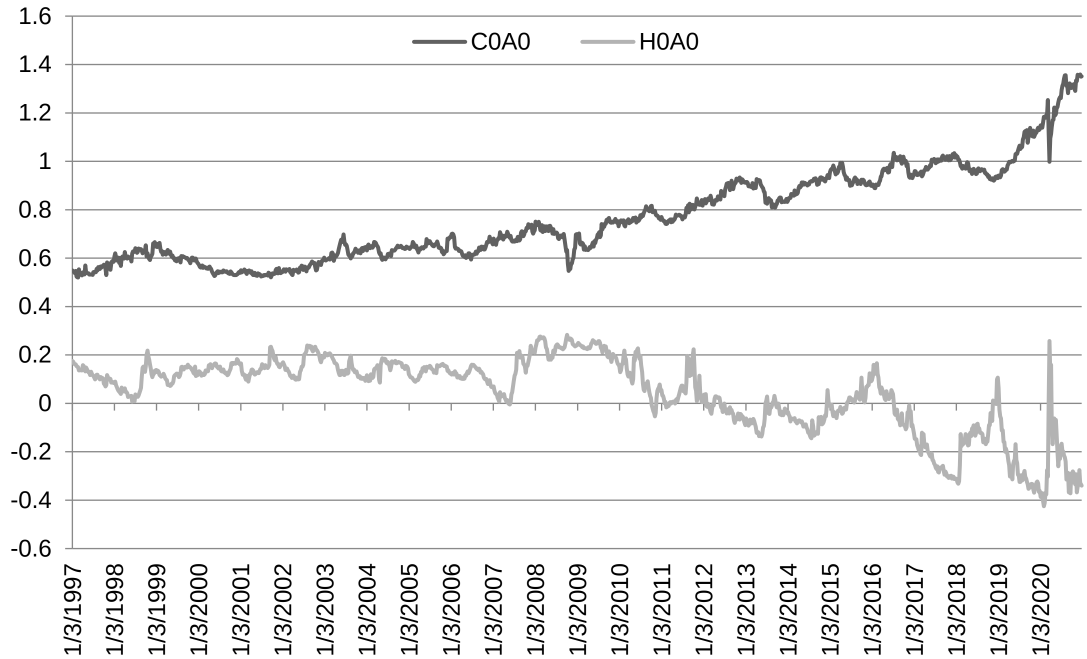
When yields move about 1 percentage point higher or 1 percentage point lower, the change in modified duration enables us to estimate the convexity of the bond. When rates increase 1 percentage point, modified duration falls by 0.251, and when rates decrease by the same amount, modified duration increases by 0.281. The average of these two values is our finite difference estimate of convexity 0.266. The convexity of a normal bond can also be calculated through the following formula.

$$\text{Convexity} = \frac{1}{P \times (1+y)^2} \times \sum_{t=1}^{T} \left[\frac{CF_t}{(1+y)^2} \times (t^2 + 1)\right] \tag{3}$$

P = Bond Price
y = Yield to Maturity
T = Maturity in t years
CF_t = Cash flow at time t

There are bond features that might interfere with the preceding duration and convexity calculations. If a bond is callable, there is a chance that the issuer will pay off the bond prior to maturity if interest rates fall and enable refinancing at lower rates. This can cause convexity to be reduced or even go negative. A call prior to maturity is much more common among high yield straight bonds than it is in investment grade straight bonds, and as a result, for the last few years through 2020, high yield corporates have on average had negative convexity, whereas investment grade corporates have enjoyed positive convexity.

C12.C5.F5.01 High Yield and Investment Grade Convexity



Data: Bloomberg

Convertibles typically have lower coupons than straight bonds, which is the tradeoff for having the embedded optionality. The lower coupon increases the rate sensitivity of the bond portion of a convertible. If the coupon in our example were 2% rather than 3%, such a convertible would have a rho of 6.02—which would be higher than the rho with a 3% coupon, but would still be only 78% of the rho of the straight bond.

C12.S5.1.3 Spread Duration (Omicron) and Spread Convexity (q_{spread})

For straight bonds, a change in yield—whether it is due to a change in rates or a change in credit spreads—has the same effect if the change in yield is measured in basis points. In other words, if yield increases 10 basis points due to an upward shift in the risk-free curve of 10 basis points, or if yield increases 10 basis points because of a 10 basis points widening of the credit spread, the change in the value of the bond is the same.

However, duration can be expressed in terms of a basis points change in yields, or as a percentage points change.

Let’s consider a 1% increase in rates and a 1% increase in credit spread for a bond that yields 5% and has a credit spread of 300 basis points over the risk-free yield of 200 basis points. A 1% increase in spreads means that spreads are increasing 3 bp, but a 1% increase in rates means that rates are increasing 2 bp. The 1% increase in spreads therefore has a greater effect on the yield of the bond than the 1% increase in interest rates. Hence one would expect spread duration (as measured on a percentage basis) to be higher than rate duration (as measured on a percentage basis). This effect is carried through to spread convexity and rate convexity.

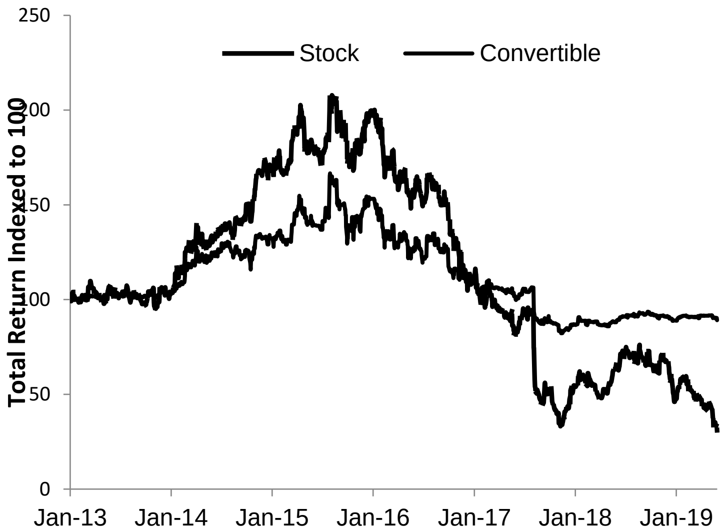
With convertible bonds, however, spread duration and rate duration differ for one additional reason. While both credit spreads and rates affect the value of the bond portion, only rates influence the value of the call option (unless the credit-risk rate is being used to value the option). Remember our option rho formula used the risk-free rate in its calculations, not the actual market rate of the bond. This means that an increase in credit spreads will cause a larger decrease in price than an equivalent increase in rates. To return to our example, the rho of the convertible under a change in credit spread is

$$\rho_{\text{convertible}} = \left(\frac{845.57}{1000} \times 8.245\right) + \left(\frac{154.43}{1000} \times -0\right) = 6.972$$

Another important consideration is what portion of the convertible value is attributable to the option, and what portion is attributable to the bond. Our reader may have noticed that the final step for calculating the rho of a convertible comes in the form of a weighted average of the two pieces. For convertibles that are more in the money, the option represents a greater percentage of the value of the convertible as the negative rate duration and zero spread duration of the option become a bigger percentage of the convertible, which pushes overall duration down. For more out of the money convertibles, the option becomes worth relatively little, the convertible becomes more bond-like and the positive durations of the bond dominate. (If a convertible bond becomes distressed, however, the influence of duration will fade, the influence of credit spread will increase, and ultimately the bond will become more equity-like if the market anticipates a cents-on-the-dollar recovery of the face value of the bond in a bankruptcy or other financial restructuring. It is extremely difficult to model such situations quantitatively.)

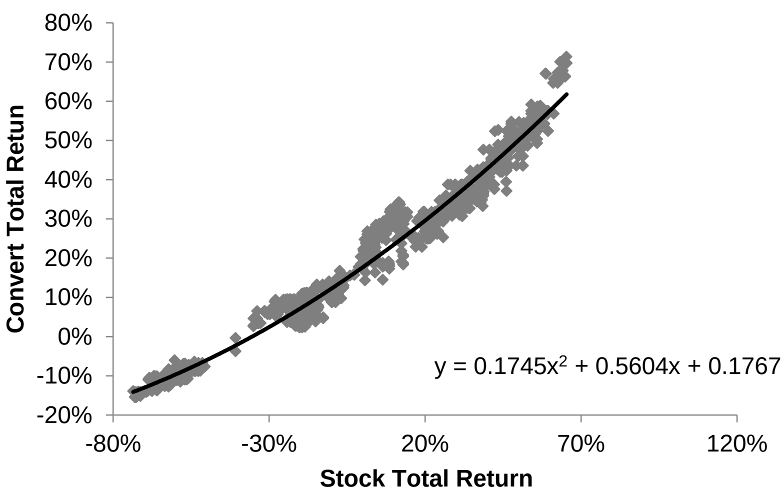
C12.S5.2 HISTORICAL EXAMPLE

C12.S5.F5.02 Timeline of Performance (Total Return) of Teva 0.25% Convertible 2026 in 2013-2019



Data: Bloomberg

C12.S5.F5.03 Chart of Total Return of Convertible vs Total Return of Stock in 2013-2019



Data: Bloomberg

In 2006 generic drug producer Teva issued a $575 million 0.25% 2026 (puttable on February 1, 2021) convertible bond and a $750 million 1.75% convertible bond, also with a 2026 maturity. The two bonds were issued in order to refinance short term debt associated with Teva's acquisition of IVAX Corporation. The 1.75% convertible bond was called for redemption in 2011 but the 0.25% convertible remained outstanding until it was put back to the company on February 1, 2021. (With a coupon payment as low as 0.25% it’s easy to see why Teva did not call the bond).

In C12.S5.F5.02 and C12.S5.F5.03 we are focused on the time period from the start of 2013 to 2019 which tracks the rise and then collapse of Teva’s stock price. Teva initially benefited, as did the generic drug industry as a whole, from the increased demand for generic drugs as a method of controlling rising healthcare spending. Then Teva acquired Allergan for a hefty $40.5 billion (nearly $34 billion of which was cash) in 2015. This left Teva with a leveraged balance sheet and ill-equipped to face accelerating generic drug price deflation that began shortly thereafter.

For our purposes it is interesting to consider what happened to the rho of the convertible over this time period using what we have learned about how convertible interest rate exposure works. In 2015, after Teva’s share price had doubled since the start of 2013, much of the convertible’s value was coming from the option value associated with the conversion feature. As we have seen the lower rho of convertible bonds comes from the counteracting force of the option value and from the fact that the bond value is only a portion of the total value of the convertible.

So in 2015, when Teva’s stock price (and therefore the conversion value as well) was at its peak, the rho of Teva’s convertible bond was very low, potentially even zero. Later on, when it became clear that Teva had overpaid for Allergan, Teva shares collapsed 80%, wiping out the option value of the bond in the process (though it appears that Teva was not considered a major credit risk despite the business problems, because the bond floor held). By 2017 the chances of Teva recovering to the conversion price had become remote, and most of the convertible’s value came from the bond component, so the convertible’s rho was nearly the same as the rho of a straight bond. Notably, the convertible included a par put in February 2021—when the out-of-the-money bond was redeemed. The overarching point of this example is that interest rate sensitivity is not a “set and forget” characteristic. The rho of a convertible security (or a portfolio of convertible securities) can change dramatically based on the performance of the underlying equity.